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EXPONENTS IN LIFETIME AND POWER SPECTRAL DENSITY FORMS IN SELF-ORGANIZED CRITICAL SYSTEMS

L.V. MEISEL

P.J. COTE

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US ARMY ARMAMENT RESEARCH, DEVELOPMENT AND ENGINEERING CENTER

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13. ABSTRACT *(Maximum 200 words)*
Bak, Tang, and Weisenfeld (BTW) established that power-law frequency dependencies in the power spectral density (PSD) and size-effect modified power-law distributions of lifetimes are the fingerprints of self-organized critical systems. Jensen, Christensen, and Fogedby (JCF) clarified the ideas introduced by BTW and established the connection between the distribution of lifetimes and the PSD for the case of exponentially cutoff ("size-effect" modified) distributions of lifetimes. Here the (JCF) connection between the PSD and the distribution of lifetimes is established for sharp cutoff distributions, which supports the idea that the JCF connection holds for quite general "size-effect" modified lifetime distributions. The PSD may be expressed in terms of generalized hypergeometric functions in this case. A detailed discussion of the JCF connections is presented for a subset of values of the lifetime distribution exponent for which the generalized hypergeometric functions reduce to Fresnel integrals and sine and cosine integrals, which were the subject of a recent "Numerical Recipes" column. All calculations were performed in Mathematica.

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BACKGROUND OF THE JCF RULE

Bak, Tang, and Weisenfeld (BTW) (ref 1) introduced the concept of self-organized critically (SOC) to provide a consistent explanation for the fractal spatial structures, power-law distributions, and flicker noise commonly observed in spatially extended, dissipative, dynamical systems. Jensen, Christensen, and Fogedby (JCF) (ref 2) clarified the ideas in BTW and established the connection between the power-law dependencies of the distribution of lifetimes and the power spectral densities (PSD).

Denoting the lifetime of an event by *T*, the JCF weighted distribution of lifetimes *G(T)* is defined as

$$G(T) = \int_0^\infty dS\, P(S,T) \left(\frac{S}{T}\right)^2 \tag{1}$$

where *P(S,T)* is the joint probability for total time integrated "sliding" *S* and lifetime *T* of an event (e.g., an avalanche). JCF demonstrate that the PSD corresponding to such a weighted distribution of lifetimes is given by

$$S(f) = \frac{v}{(\pi f)^2} \int_0^\infty dr\, G(r) \sin^2(\pi f r) \tag{2}$$

where v is the pulse repetition rate and f is the frequency.

JCF assumed that the weighted distribution of lifetimes *G(T)* varies (approximately) as

$$G(T) \propto \begin{cases} 0, & \text{when } T < t_0 \\ T^{\alpha}\exp(-T/T_0), & \text{when} T \geq t_0. \end{cases} \tag{3}$$

We refer to the form of Eq. (3) as an "exponentially cutoff power-law distribution of lifetimes." The parameters in the distribution are referred to as: (1) the lifetime distribution exponent: α; (2) the exponential cutoff parameter: T_0; and (3) the lower lifetime cutoff: t_0.

JCF established that an exponentially cutoff power-law distribution of lifetimes gives rise to PSD *S(f)* of the form,

$$S(f) \propto \begin{cases} const, & \text{when } f < 1/T_0, \\ f^E, & \text{when } 1/T_0 \leq f \leq 1/t_0, \\ f^{-2}, & \text{when } f > 1/t_0. \end{cases} \tag{4}$$

We refer to the exponent *E* as the power spectral density exponent. Further, JCF established that the power spectral density exponent is determined by its counterpart in the distribution of lifetimes, viz.,

$$E = \begin{cases} 0, & \text{when } \alpha < -3, \\ -(3+\alpha), & \text{when } -3 \le \alpha < -1, \\ -2, & \text{when } \alpha \ge -1. \end{cases} \tag{5}$$

Note the appearance of critical frequencies $1/T_0$ and $1/t_0$ at which the frequency dependence of the PSD changes form. Equations (4) and (5) constitute the JCF connection between the distribution of lifetimes and the PSD.

The JCF connection pertains to a wider range of systems than those resulting from SOC processes. The results are consequences of the absence of characteristic length and time scales and therefore apply to systems that exhibit fractal scaling, etc., independent of an organizing principle.

Furthermore, the JCF connection does not depend on the specific form of the cutoff power-law distribution of lifetimes. This is important because the data are not necessarily well-described by exponentially cutoff power laws. For example, the behavior of real sandpiles (ref 3) and the Barkhausen effect (ref 4) indicate that the distributions are cut off, but the best form is not obvious. The distribution of lifetimes in the Barkhausen effect in three ferromagnetic metals (Metglas 2605S ($Fe_{78}B_{13}Si_{19}$), polycrystalline iron thermocouple wire, and Alumel ($Ni_{95}A_3Mn_2$)) was determined to be better represented as sharp cutoff than exponentially cutoff weighted distributions of lifetimes in Reference 4.

In this report we demonstrate that a sharp cutoff weighted distribution of lifetimes gives rise to the JCF parameter connections with the cutoff lifetime (largest lifetime in a "size effect" limited distribution) playing the role of T_0 in the exponentially cutoff distribution, as was claimed in Reference 4.

THE SHARP CUTOFF FORM

Assume that the weighted distribution of lifetimes $G(T)$ varies (approximately) as a sharp cutoff power law

$$G(T) \propto \begin{cases} 0, & \text{when } T < t_0 \\ T^{\alpha}, & \text{when } T_0 \ge T \ge t_0 \\ 0, & \text{when } T > T_0 \end{cases} \tag{6}$$

so that from Eq. (2), the PSD is given by

$$S(f) = \nu \int_{t_0}^{T_0} d\tau \, \tau^{\alpha} \left(\frac{\sin(\pi f \tau)}{\pi f} \right)^2 \tag{7a}$$

$$= \nu (\pi f)^{-(\alpha+3)} \int_{\pi f t_0}^{\pi f T_0} dx \, x^{\alpha} (\sin^2(x)) \tag{7b}$$

$$= \nu (\pi f)^{-(\alpha+3)} W(\alpha, \pi f t_0, \pi f T_0) \tag{7c}$$

where Eq. (7c) serves to define the function W

$$W(\alpha,a,b) = \int_a^b dx \, x^{\alpha} \sin^2(x) \tag{7d}$$

One sees that $S(f)$ is proportional to $f^{(\alpha+3)} W(\alpha, \pi f t_0, \pi f T_0)$.

ANALYTIC RESULTS

General Solution of the Sharp Cutoff JCF Problem

Mathematica is well-suited to the evaluation of the sharp cutoff expressions for the PSD. For general values of α, $W(\alpha,a,b)$ may be expressed in terms of generalized hypergeometric functions ${}_pF_q(z)$. After some editing, the Mathematica input

Integrate[x^alpha*Sin*[x]^2,{x,a,b}] yields:

$$W(\alpha,a,b) = \frac{b^{1+\alpha}}{2(1+\alpha)} (1 - {}_{\{\frac{1}{2}+\frac{\alpha}{2}\}}F_{\{\frac{1}{2},\frac{3}{2}+\frac{\alpha}{2}\}}(-b^2)) - \frac{a^{1+\alpha}}{2(1+\alpha)} (1 - {}_{\{\frac{1}{2}+\frac{\alpha}{2}\}}F_{\{\frac{1}{2},\frac{3}{2}+\frac{\alpha}{3}\}}(-a^2))$$

It is clear that one can express the general α form of $W(\alpha,a,b)$ in terms of functions dependent on the limits separately, i.e.,

$$W(\alpha,a,b) = w(\alpha,b) - w(\alpha,a) \tag{8a}$$

$$w(\alpha,t) = \frac{t^{1+\alpha}}{2(1+\alpha)} (1 - {}_{\{\frac{1}{2}+\frac{\alpha}{2}\}}F_{\{\frac{1}{2},\frac{3}{2}+\frac{\alpha}{2}\}}(-t^2)) \tag{8b}$$

Mathematica 2.1 evaluates the generalized hypergeometric function

$$_{\left\{\frac{1}{2}+\frac{\alpha}{2}\right\}}F_{\left\{\frac{1}{2},\frac{3}{2}+\frac{\alpha}{2}\right\}}(-x^2) \textit{ for } x < 20$$

but fails for larger values of x. (Mathematica 2.1 is employed to obtain *numerical values* of $W(\alpha,a,b)$ for arbitrary α, a, and b via NIntegrate[x^alpha *Sin*[x]^2,{x,a,b}].)

Special values of α

When α is integer or half-integer valued, W may be expressed in terms of Fresnel integrals and cosine and sine integrals, a better known set of functions, which were recently the subject of a "Numerical Recipes" column of Press and Teukolsky (ref 5).

Employing the Mathematica code:

```
For[i = -8, i < 3, i + +,

    w[i/2,t_]:=Evaluate[

        Map[Simplify[Apart[#]]&,Integrate[t^(i/2)Sin[t]^2,t]]]];
```

one obtains the following expressions $w[\alpha,t]$ for integer and half-integer α values in the range of interest

$$w[-4,t] = \frac{-1}{6t^3} + \frac{(1-2t^2)\cos(2t)}{6t^3} - \frac{\sin(2t)}{6t^2} - \frac{2\,SinIntegral(2t)}{3} \tag{9a}$$

$$w[-7/2,t] = \frac{-1}{5t^{\frac{5}{2}}} + \frac{(3-16t^2)\cos(2t)}{15t^{\frac{5}{2}}} - \frac{32\sqrt{\pi}\,FresnelS\left(\frac{2\sqrt{t}}{\sqrt{\pi}}\right)}{15} - \frac{4\sin(2t)}{15t^{\frac{3}{2}}} \tag{9b}$$

$$w[-3,t] = \frac{-1}{4t^2} + \frac{\cos(2t)}{4t^2} + CosIntegral(2t) - \frac{\sin(2t)}{2t} \tag{9c}$$

$$w[-5/2,t] = \frac{-1}{3t^{\frac{3}{2}}} + \frac{\cos(2t)}{3t^{\frac{3}{2}}} + \frac{8\sqrt{\pi}\,FresnelC\left(\frac{2\sqrt{t}}{\sqrt{\pi}}\right)}{3} - \frac{4\sin(2t)}{3\sqrt{t}} \tag{9d}$$

$$w[-2,t] = \frac{-1}{2t} + \frac{\cos(2t)}{2t} + SinIntegral(2t) \quad \textbf{(9e)}$$

$$w[-3/2,t] = -\frac{1}{\sqrt{t}} + \frac{\cos(2t)}{\sqrt{t}} + 2\sqrt{\pi}\ FresnelS\left(\frac{2\sqrt{t}}{\sqrt{\pi}}\right) \quad \textbf{(9f)}$$

$$w[-1,t] = \frac{-CosIntegral(2t)}{2} + \frac{\log(t)}{2} \quad \textbf{(9g)}$$

$$w[-1/2,t] = \frac{\sqrt{\pi}\left(\frac{2\sqrt{t}}{\sqrt{\pi}} - FresnelC\left(\frac{2\sqrt{t}}{\sqrt{\pi}}\right)\right)}{2} \quad \textbf{(9h)}$$

$$w[0,t] = \frac{t}{2} - \frac{\sin(2t)}{4} \quad \textbf{(9i)}$$

$$w[1/2,t] = \frac{t^{\frac{3}{2}}}{3} + \frac{\sqrt{\pi}\,FresnelS\left(\frac{2\sqrt{t}}{\sqrt{\pi}}\right)}{8} - \frac{\sqrt{t}\sin(2t)}{4} \quad \textbf{(9j)}$$

$$w[1,t] = \frac{t^2}{4} - \frac{\cos(2t)}{8} - \frac{t\sin(2t)}{4} \quad \textbf{(9k)}$$

With these analytic expressions, one can consider various ranges of f and demonstrate that the JCF connections obtain for the special α values. We consider a few cases in detail:

A. $\alpha = -2$.

1. The behavior of $w[-2,t]$ given in Eq. (9e).

a. Small t. One could use the expansion of Eq. (16) of Reference 5, etc., to obtain the small t form of $w[-2,t]$. Here we employ the Series command from Mathematica to obtain

$$w[-2,t] \rightarrow t - \frac{t^3}{9} + \frac{2t^5}{225} - \frac{t^7}{2205} + O(t)^8$$

b. Large t. Using the large t limiting form (which can be deduced in Mathematica):

$$SinIntegral[t] + \frac{\cos t}{t} \to \pi/2$$

one obtains:

$$w[-2,t] \to -\frac{1}{2t} + \frac{\pi}{2} \to \frac{\pi}{2}$$

2. The behavior $W[-2,a,b]$ and the PSD exponent. From Eq. (7c),

$$S(f) \propto f^{-(\alpha+3)} W(\alpha,\pi f t_0,\pi f T_0) \to f^{-1} W(-2,\pi f t_0,\pi f T_0)$$

a. $a << b << 1$ or $f << 1/T_0 << 1/t_0$.

$$W[-2,a,b] \approx w[-2,b] \to \pi f T_0 \; and \; S(f) \to const$$

b. $a << 1 << b$ or $1/T_0 << f << 1/t_0$.

$$W[-2,a,b] \approx w[-2,b] \to \pi/2 \; and \; S(f) \propto f^{-1}$$

c. $a << b << 1$ or $1/T_0 << 1/t_0 << f$.

The $\pi/2$ terms in $w[-2,a]$ and $w[-2,b]$ cancel and

$$W[-2,a,b] \approx \frac{1}{2a} - \frac{1}{2b} \approx \frac{1}{2\pi f t_0}$$

Thus,

$$S(f) \propto f^{-1} \times 1/f = f^{-2}$$

Thus, the JCF connections are established for the case $\alpha = -2$.

B. $\alpha = -3/2$.

1. The behavior of $w[-3/2,t]$ given in Eq. (9f).

a. Small t. One could use the expansion of Eq. (10) of Reference 5, etc., to obtain the small t form of $w[-3/2,t]$. Here we employ the Series command from Mathematica to obtain:

$$w[-3/2,t] \rightarrow \frac{2t^{\frac{3}{2}}}{3} - \frac{2t^{\frac{7}{2}}}{21} + \frac{4t^{\frac{11}{2}}}{495} + O(t)^{\frac{13}{2}}$$

b. Large t. Using the large t limiting form of Eq. (15) of Reference 5, one obtains:

$$w[-3/2,t] \rightarrow -\frac{1}{\sqrt{t}} + \sqrt{\pi} \rightarrow \sqrt{\pi}$$

2. The behavior $W[-3/2,a,b]$ and the PSD exponent. From Eq. (7c)

$$S(f) \propto f^{-(\alpha+3)}W(\alpha,\pi f t_0,\pi f T_0) \rightarrow f^{-3/2}W(-3/2,\pi f t_0,\pi f T_0)$$

a. $a << b << 1$ or $f << 1/T_0 << 1/t_0$.

$$W[-3/2,a,b] \approx w[-3/2,b] \rightarrow \frac{2}{3}(\pi f T_0)^{3/2} \textit{ and } S(f) \rightarrow const$$

b. $a << 1 << b$ or $1/T_0 << f << 1/t_0$.

$$W[-3/2,a,b] \approx w[-3/2,b] \rightarrow \sqrt{\pi} \textit{ and } S(f) \propto f^{-3/2}$$

c. $a << b << 1$ or $1/T_0 << 1/t_0 << f$.

The $\sqrt{\pi}$ terms in $w[-3/2,a]$ and $w[-3/2,b]$ cancel and

$$W[-3/2,a,b] \approx \frac{1}{\sqrt{a}} - \frac{1}{\sqrt{b}} \approx 1/\sqrt{\pi f t_0}$$

Thus,

$$S(f) \propto f^{-3/2} \times f^{-1/2} \propto f^{-2}$$

Thus, the JCF connections are established for the case $\alpha = -3/2$.

C. $\alpha = 1$.

1. The behavior of $w[1,t]$ given in Eq. (9k).

a. Small t. Employ the Series command from Mathematica to obtain

$$w[1,t] \rightarrow -\frac{1}{8} + \frac{t^4}{4} - \frac{t^6}{18} + O(t)^8$$

b. Large t.

$$w[1,t] \rightarrow \frac{t^2}{4}$$

2. The behavior $W[1,a,b]$ and the PSD exponent. From Eq. (7c),

$$S(f) \propto f^{-(\alpha+3)} W(\alpha,\pi f t_0,\pi f T_0) \rightarrow f^{-4} W(1,\pi f t_0,\pi f T_0)$$

a. $a << b << 1$ or $f << 1/T_0 << 1/t_0$. The 1/8 terms in $w[1,a]$ and $w[1,b]$ cancel and

$$W[1,a,b] \approx w[1,b] \rightarrow (\pi f T_0)^4/4 \text{ and } S(f) \propto f^{-4} \times f^4 \rightarrow const$$

b. $1 << b$ or $1/T_0 << f$ (both subranges included).

$$W[1,a,b] \approx w[1,b] \rightarrow t^2/4 \text{ and } S(f) \propto f^{-4} \times f^2 \rightarrow f^{-2}$$

Thus, the JCF connections are established for the case $\alpha = 1$.

Similar analysis can be applied for all the forms in Eq. (9) and for integer and half-integer α generally.

NUMERICAL RESULTS

The analytic results of Eqs. (7) through (9) can be used to compute $S(\pi f T_0)$ for specific values of α and T_0/t_0. Figure 1 presents typical PSD versus frequency results, which were obtained for

$$T_0 = e^{12} \times t_0$$

and $\alpha \in \{-7/2, -5/2, -2, -3/2, 1\}$. The value of α increases from -7/2 for the top curve to +1 for the bottom curve. The vertical line at $\ln(\pi f T_0) = 12$ corresponds to

$$\ln(\pi f t_0) = \ln(\pi f T_0) - 12 \rightarrow 12 - 12 = 0$$

Thus the breaks in slope occur for

$$\ln(\pi f t_0) = 0 \quad and \quad \ln(\pi f T_0) = 0$$

as advertised. All curves become f-independent for $\ln(\pi f T_0) < 0$ and exhibit inverse square PSD for $\ln(\pi f t_0) > 0$. The top curve is typical of $\alpha < -3$ cases. The three middle curves represent the $-3 \leq \alpha \leq -1$ range for which $E = -(3+\alpha)$; the variations in slope are apparent. The lowest curve is typical of the $\alpha > -1$ range. Although it is unlikely that one could observe such effects in actual PSD curves, the "bumpiness" to the right of the high frequency transitions is real (i.e., not numerical).

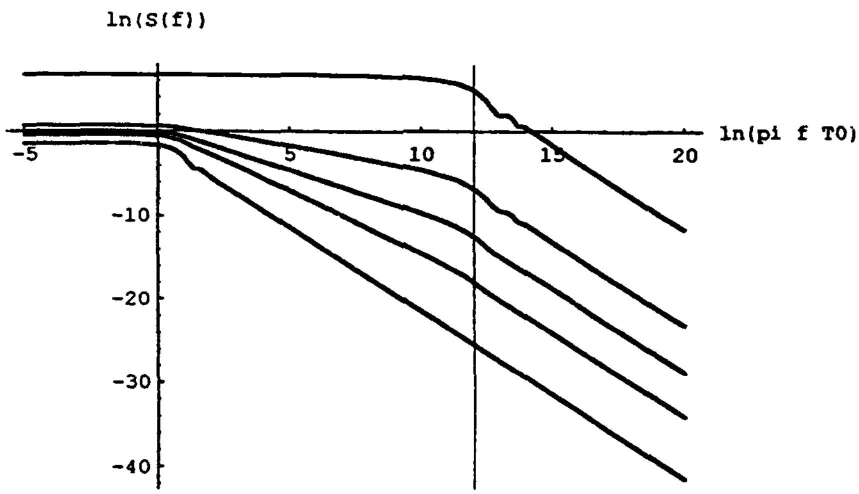

Figure 1. Power spectral density versus frequency for $T_0 = e^{12} \times t_0$. Curves are shown for lifetime distribution exponent $\alpha \in \{-7/2, -5/2, -2, -3/2, 1\}$. α increases from -7/2 for the top curve to +1 for the bottom curve. The vertical line at $\ln(\pi f T_0) = 12$ corresponds to $\ln(\pi f t_0) = 0$.

Results obtained via NIntegrate at arbitrary α fall neatly between the curves shown in Figure 1 and those for half-integer and integer α are indistinguishable from curves obtained from the analytic expressions.

CONCLUSIONS

The connection between the distribution of lifetimes and the PSD established by Jensen, Christensen, and Fogedby (ref 2) for the case of exponentially cutoff distributions of lifetimes has been shown to apply, with natural parameter correspondences, to sharp cutoff distributions of lifetimes. Since the range of cutoff forms between exponential and sharp is broad, the present results suggest that the JCF connections will obtain to a very wide range of size-effect modified, self-organized critical systems.

The power of symbolic computational systems, such as Mathematica, is nicely illustrated by the present analysis. As presently described, all calculations can be achieved in Mathematica. One could also employ the analytic results as starting points for other (e.g., Fortran or C) computer programs.

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